1 Introduction

This document serves as a user guide for the Memristor Compact model initially proposed in [1]. The proposed model is validated against experimental data of HfO$_x$ devices fabricated in-house at SUNY Polytechnic Institute [2]. While the model can be generally applied to any memristive device with sufficient accuracy, its structure and parameters are ideally suited for Transition Metal Oxide (TMO) (i.e. RRAM) devices. Several models have been developed since 2008 that vary from physics-based compact models that are specific to some particular switching mechanism to behavioral models that hinge on the fundamental memristor equations proposed by Chua.

The difficulty in modeling and understanding memristive dynamics rests in their dynamic behavior which circuit designers are not accustomed to. Most memristor models rely on defining a state variable controlled by either voltage or current. The state variable can be a physical quantity such as the gap distance in physical models or abstract quantity in behavioral models. The state variable is then translated into resistance via an equation which similar to the state variable might describe a physical phenomenon such as tunneling in physical models or a behavioral equation in behavioral models.

The challenge, however, is that both approaches do not necessarily reflect measurable parameters which experimentalists report. Experimental data is usually presented in the form of V-I relationship from which the effective device resistance can be extracted. However, making the connection between the extracted resistance and the state variable, be it physical or abstract, is left to the modeler to interpret which is usually challenging!

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2 The Resistance-based approach

Here we take a different approach which uses the instantaneous resistance as the state variable. While this approach might not be appreciated by memristor "theoreticians", it is perfectly suited for circuit designers who wish to have a "plug-and-play" model which is (1) simple to understand (2) suited for parameter extraction from physical data and (3) converge well in circuit simulation.

2.1 Model Description

\[
\frac{dM}{dt} = \begin{cases} 
  -C_{LRS}(V(t) - V_{tp})P_{LRS} f_{LRS}(M(t)), & V(t) > V_{tp} \\
  C_{HRS}(V(t) - V_{tn})P_{HRS} f_{HRS}(M(t)), & V(t) < V_{tn} \\
  0, & \text{otherwise},
\end{cases}
\]

(1)

Here $C_{LRS}$ and $C_{HRS}$ are two scale parameters. $P_{LRS}$ and $P_{HRS}$ are two non-linearity coefficients. $f_{HRS}$ and $f_{LRS}$ capture the resistance saturation (commonly referred to as window functions). Equation (2) presents the proposed window function that can be easily fitted to measurable parameters.

\[
f(M(t)) = \begin{cases} 
  \frac{1}{1 + e^{\frac{M(t) - M_{LRSLRS}}{\beta_{LRSLRS}}}}, & V(t) < V_{tn} \\
  \frac{1}{1 + e^{\frac{M(t) - M_{HRS}}{\beta_{HRS}}}}, & V(t) > V_{tp}
\end{cases}
\]

(2)

In (2), $M(t)$ is still clipped to either $LRS$ or $HRS$ once the resistance hits either boundary to ensure it does not exceed the measurable range of resistance such that:

\[
If \ (M(t) > HRS), \ then \ M(t) = HRS,
\]

(3)

\[
If \ (M(t) < LRS), \ then \ M(t) = LRS,
\]

(4)
where (3) and (4) are implemented as the device is switching from \textit{LRS} to \textit{HRS} and \textit{HRS} to \textit{LRS}, respectively.

In fact, the user can opt to activate just the clipping function without applying the window function in (2) using window selectors as shown in the following section.

\section*{2.2 Using the Model}

The model has a window selector, such that:

- \textit{window} = 0: only clipping is activated as shown in (3) and (4).
- \textit{window} = 1: Window function + clipping are activated as shown in (2), (3) and (4).

The Model parameters as well as the suggested values are depicted in Table 1.

* please note that while the equations in (2) and (3) include \( C_{\text{HRS/LRS}} \) as the only scale parameter, the way it is implemented in the model is:

\[
C_{\text{HRS/LRS}} \cdot \frac{\Delta r}{t_{\text{sw}_{p/n}}},
\]

In (2) and (3) the \( \frac{\Delta r}{t_{\text{sw}_{p/n}}} \) factor was absorbed in \( C \) as a single fitting parameter.

** The threshold voltage in this model is defined as the voltage below which the change in resistance is forced to zero. In reality, there is no threshold similar to many other semiconductor devices. However, there is a point at which a significant nonlinearity is observed (more like a "knee" in the curve). These threshold voltages are to be adjusted such that the model fits the experimental data.

Fig 1 depicts the memrsitor symbol. \( p \) and \( n \) are the positive and negative terminals, respectively. \( r \) is the resistance terminal. It should be noted that \( r \) is not a physical terminal but it is only there to facilitate the computation of the instantaneous resistance. Instead of dividing the Voltage by the Current to compute the resistance, the verilog-A code computes the resistance internally and outputs it to terminal \( r \).

Fig 2 depicts the time domain response of a voltage sin wave where the initial resistance is set to \textit{HRS}. Fig 3 shows the well known hysteresis in the V-I plane as the Current response is plotted against the Voltage signal.
Figure 1: memristor symbol

Figure 2: Time domain response to a Voltage sin wave.
Figure 3: Hysteresis in the V-I plane.

References


Table 1: Suggested Model Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>Value used in the code</th>
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<tbody>
<tr>
<td>$V_{tp}$</td>
<td>+ve threshold Voltage</td>
<td>$0.7V^{**}$</td>
</tr>
<tr>
<td>$V_{tn}$</td>
<td>-ve threshold Voltage</td>
<td>$-0.7V^{**}$</td>
</tr>
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<td>$t_{sw_p}$</td>
<td>switching time, HRS to LRS</td>
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<tr>
<td>$t_{sw_n}$</td>
<td>switching time, LRS to HRS</td>
<td>$1us$</td>
</tr>
<tr>
<td>$C_{LRS}$</td>
<td>scale parameter, HRS to LRS</td>
<td>$1^*$</td>
</tr>
<tr>
<td>$C_{HRS}$</td>
<td>scale parameter, LRS to HRS</td>
<td>$1^*$</td>
</tr>
<tr>
<td>$HRS$</td>
<td>High Resistance State</td>
<td>$150K\Omega$</td>
</tr>
<tr>
<td>$LRS$</td>
<td>Low Resistance State</td>
<td>$10K\Omega$</td>
</tr>
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<td>$\Delta r$</td>
<td>$(HRS - LRS)$</td>
<td>$140K\Omega$</td>
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<td>non-linearity parameter, HRS to LRS</td>
<td>$3$</td>
</tr>
<tr>
<td>$P_{HRS}$</td>
<td>non-linearity parameter, LRS to HRS</td>
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